As a check upon the work, the identities (10)–(27) can also be obtained by direct multiplication. For example, represent

$$\begin{vmatrix} \lambda_1' & \mu_1' & \nu_1' \\ \lambda_2 & \mu_2 & \nu_2 \\ \lambda_3 & \mu_3 & \nu_3 \end{vmatrix}$$

by B_1; then, in the identity $A' \cdot (A \cdot B_1) = A \cdot (A' \cdot B_1)$, first on the left hand side multiply together the determinants A and B_1, and on the right hand side A' and B_1; the result is one of the identities (10)–(27).

The identities (10)–(27) are often useful in changing the form of the equations for the transformation from one system of oblique axes to another oblique system; for example, the equations (2) on p. 197 of Grunert, *Arch. d. Math.*, 34; also in proving the relation between the formulas here given and the formulas, for example, of Grunert and Sturm.

QUESTIONS AND DISCUSSIONS.

EDITED BY U. G. MITCHELL, University of Kansas.

NEW QUESTIONS.

25. In an investigation in physics Mr. Mason E. Hufford, 525 S. Park Ave., Bloomington, Indiana, has need of the values of the Bessel functions $J_0(x)$ and $J_1(x)$ for positive real values of x up to $x = 100$. Have tables been constructed to this extent? What is the most ready means by which the desired values may be computed to any required degree of accuracy?

REPLIES.

24. The following facts are significant:

(1) The New England Association of Mathematics Teachers has appointed a committee "to investigate the current criticisms of high school mathematics."

(2) A committee of the Council of the American Mathematical Society has under consideration the question "whether any action is desirable on the part of the Society in the matter of the movement against mathematics in the schools."

(3) At the recent meeting in Cincinnati of the National Education Association an iconoclastic discussion on the topic: "Can algebra and geometry be reorganized so as to justify their retention for high school pupils not likely to enter technical schools?" aroused approbation and applause. An outline of the remarks by one of the speakers is printed below.

In view of these facts what should be done by those who believe in the value of mathematics as a general high school study?

MEMORANDUM OF REMARKS MADE BY COMMISSIONER DAVID SNEDDEN, of Massachusetts, before the Mathematics Section of the Commission on the Reorganization of Secondary School Studies, Cincinnati, Ohio, February 25th, 1915.

Commissioner Snedden followed in his discussion the presentation of a paper from Assistant Superintendent Wicher, of New Hampshire, on the subject: "Can Algebra and Geometry be Reorganized so as to justify their Retention for Pupils Not Likely to Enter Technical Schools?" He said in substance:

(*a*) That at present algebra and geometry occupy substantially monopolistic positions in the curricula of secondary schools, in that in many such schools they

are required for graduation, and further, in that they are usually required for admission to college;

(*b*) That at the present time there is much uncertainty as to the actual educational values of these studies, more especially for girls, and that the burden of proof rests upon those who desire to have them further continued as prescriptive studies;

(*c*) That the great majority of teachers of mathematical subjects in high schools are not informed as to the ultimate educational values of these studies. They content themselves with pursuing proximate aims, among which are the mastery of the matter contained in the text-book, enabling students to meet closing examinations or college entrance examinations, etc.

(*d*) That among writers and thinkers on the subject of instruction in mathematics there seem to be five prominent ultimate aims held forth as justifying the importance attached to these studies in secondary schools, namely:

1. The so-called "disciplinary aim."
2. The so-called "instrumental aim," for students going into engineering and other callings requiring advanced mathematical knowledge.
3. The other so-called "instrumental aim," for students pursuing advanced studies in such fields as physics, economics, etc.
4. The first so-called "cultural aim," in which it is held that mathematical studies serve to interpret for the student the world in which he lives.
5. The second so-called "cultural aim," to the effect that the study of mathematics enables the student to interpret a substantial part of his social inheritance.

(*e*) That of the foregoing aims, that which has the largest support is the so-called "disciplinary aim." The value of mathematical studies under this head, however, has been seriously called into question by recent investigations, and the majority of the students of education are now inclined to attach relatively little value to it, holding that mental discipline must be a by-product of any and all studies pursued because of their ultimate worth in fitting the student for life in its cultural, civic and vocational aspects.

(*f*) That second in importance as the aim of mathematics, in the opinion of writers and speakers, is the claim that it serves in an instrumental capacity both for vocations and for subsequent higher studies. It is believed that the importance of this aim has been greatly exaggerated, and that the whole subject deserves special study, particularly as to the needs of girls.

(*g*) That each of the so-called "cultural" aims is important, but it is questionable whether they are in any substantial degree realized by present methods of teaching, it being contended that present methods of teaching, including thereunder the scope and character of material used, as found in text-books, etc., have been dictated by the requirements of the instrumental and disciplinary aims, and that as a consequence the large majority of high school students, as the outcome of their study of algebra and geometry, have gained neither in comprehension of the world in which they live nor in substantial appreciation of the mathematical portion of the so-called "social inheritance."

(*h*) That the most needed reform in the teaching of mathematics in secondary schools at the present time is an analytical consideration of the subject from the standpoint, first, of its instrumental or vocational use, and, second, its importance as an agency of cultural education. The high school (and equally the college) should then offer as electives quite technical courses in mathematics as an instrumental study, to those desiring the same. These schools should also offer, probably as electives, distinctly cultural courses designed to acquaint young people with the evolution of mathematical studies and the part they now play in the life of the individual and of society, these studies to be much more descriptive and interpretive than anything now found under the heads of algebra and geometry, even including the more historical treatment of these subjects.

DISCUSSIONS.

Relating to the use of i and $e^{\phi i}$ in vector-notation.

(Cf. article "A Note on Plane Kinematics," by Alexander Ziwet and Peter Field, this MONTHLY, Vol. XXI, pp. 105–113.)

DISCUSSION BY EDWIN BIDWELL WILSON, Massachusetts Institute of Technology.

In the MONTHLY, April, 1914, Ziwet and Field gave a very interesting and exceedingly neat treatment of certain elementary properties of plane kinematics based on the notation of Burali-Forti and Marcolongo, and they interpreted the analysis in a series of fundamental constructions. The treatment is elegant not alone by what it does, but particularly by what it leaves unsaid. The fundamental operation is multiplication by i or by $e^{\phi i}$, to turn a vector through the angle $\frac{1}{2}\pi$ or φ respectively. It is not mentioned, except once parenthetically, that, used in this sense, i and $e^{\phi i}$ are very different from the ordinary $i = \sqrt{-1}$ and $e^{\phi i} = \cos\varphi + \sqrt{-1}\sin\varphi$ of algebra. Indeed it is only when these operators are applied successively to the same vector that the analogy with complex numbers is valid. When applied in products of vectors the operators are not at all scalar. This may be seen in the following parallel columns, where I use the notations $\times$ and $\wedge$ for scalar and vector products (instead of the respective $\cdot$ and $\times$ of Gibbs).

i and $e^{\phi i}$ as operators	$i = \sqrt{-1}$, $e^{\phi i} = \cos\varphi + \sqrt{-1}\sin\varphi$
$i\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{a} \times i\boldsymbol{a} = 0$	$i\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{a} \times i\boldsymbol{a} = i\boldsymbol{a}^2$
$i\boldsymbol{a} \wedge \boldsymbol{a} = -\boldsymbol{a} \wedge i\boldsymbol{a} \neq 0$	$i\boldsymbol{a} \wedge \boldsymbol{a} = \boldsymbol{a} \wedge i\boldsymbol{a} = 0$
$i\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{a} \times i\boldsymbol{b}$	$i\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{a} \times i\boldsymbol{b}$
$i\boldsymbol{a} \times i\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$	$i\boldsymbol{a} \times i\boldsymbol{b} = -\boldsymbol{a} \times b$
$e^{i\phi}\boldsymbol{a} \times e^{i\phi}\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$	$e^{i\phi}\boldsymbol{a} \times e^{i\phi}\boldsymbol{b} = e^{2i\phi}\boldsymbol{a} \times \boldsymbol{b}$
$\boldsymbol{a} \times e^{i\phi}\boldsymbol{b} = \boldsymbol{b} \times e^{-i\phi}\boldsymbol{a}$	$\boldsymbol{a} \times e^{i\phi}\boldsymbol{b} = \boldsymbol{b} \times e^{i\phi}\boldsymbol{a}$
$e^{i\phi}\boldsymbol{a} \times e^{i\psi}\boldsymbol{a} = \boldsymbol{a}^2 \cos(\psi - \varphi)$	$e^{i\phi}\boldsymbol{a} \times e^{i\psi}\boldsymbol{a} = e^{i(\phi+\psi)}\boldsymbol{a}^2$
$e^{i\phi}\boldsymbol{a} \wedge e^{i\psi}\boldsymbol{a} = \boldsymbol{a}^2 \sin(\psi - \varphi)\boldsymbol{u}$	$e^{i\phi}\boldsymbol{a} \wedge e^{i\psi}\boldsymbol{a} = 0.$

In the last row $\boldsymbol{u}$ is a unit vector perpendicular to the plane we are considering.